The Effect of Age-related Microstructural Changes on Constitutive Behavior of Human Renal Arteries

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INTRODUCTION: Arterial mechanical behavior changes throughout a person's life. Functional changes are due to microstructural changes in the arterial wall. Mathematical descriptions of the stress-strain relationships in healthy arterys have been developed [1]; however, little attention has been paid to changes in the stress-strain behavior in aging arteries. The purpose of this study was to apply a constitutive model to examine relationships between age-related changes in mechanical behavior and microstructure of human renal arteries.

METHODS: The mathematical description of the stress-stretch relationship for aging human renal arteries was derived from Fung's 2-D model [2]. The stress-strain relationship can be expressed by:

$$\left\{ t_{\theta\theta} = -p + \lambda_\theta^2 \frac{\partial W}{\partial E_{\theta\theta}},\ and\ t_{zz} = -p + \lambda_z^2 \frac{\partial W}{\partial E_{zz}}, \right. \quad (1)$$

where $E_{\theta\theta} = ½(\lambda_\theta^2 - 1)$, and $E_{zz} = ½(\lambda_z^2 - 1)$. The strain energy function, W, is given by:

$$W = \frac{c}{2}\left\{e^Q - 1\right\} \quad \text{where} \quad Q = C_1 E_{\theta\theta}^2 + C_2 E_{zz}^2 + 2C_3 E_{\theta\theta} E_{zz}$$

For uni-axial loading in the circumferential (θ) direction, $t_{\theta\theta} \neq 0$, $t_{zz} = 0$. By assuming $\lambda_z = \alpha(1 - \lambda_\theta) + 1$, equation (1) can be rewritten as:

$$Q = \frac{1}{4}(C_1 + \alpha^4 C_2 + 2\alpha^2 C_3)\lambda_\theta^4 - \alpha(\alpha+1)(\alpha^2 C_2 + C_3)\lambda_\theta^3 + \frac{1}{2}\left[-C_1 + \alpha^2(3\alpha^2 + 6\alpha + 2)C_2 + 2\alpha C_3\right]\lambda_\theta^2$$
$$+ \alpha(\alpha+1)\left[-\alpha(\alpha+2)C_2 + C_3\right]\lambda_\theta + \frac{1}{4}\left[C_1 + \alpha^2(\alpha+2)^2 C_2 - 2\alpha(\alpha+2)C_3\right]$$

where

$$t_{\theta\theta} = c\left\{\frac{1}{2}(C_1 - \alpha^4 C_2)\lambda_\theta^4 + 2\alpha^3(\alpha+1)C_2\lambda_\theta^3 + \left[-\frac{1}{2}C_1 - \frac{\alpha^2}{2}(6\alpha^2 + 12\alpha + 5)C_2 + \frac{1}{2}(\alpha^2 - 1)C_3\right]\lambda_\theta^2\right.$$
$$\left. + \left[\alpha(\alpha+1)(2\alpha^2 + 4\alpha + 1)C_2 - \alpha(\alpha+1)C_3\right]\lambda_\theta + \left[-\frac{\alpha}{2}(\alpha+1)^2(\alpha+2)C_2 + \frac{1}{2}(\alpha+1)^2 C_3\right]\right\} \cdot e^Q$$

The stress-stretch relationship of uni-axial loading in the z direction can be obtained by assuming $\lambda_\theta = \beta(1 - \lambda_z) + 1$.

The model has six parameters, α, β, c, C_1, C_2, and C_3. α and β are given by $\alpha = \Delta\lambda_\theta / \Delta\lambda_z$, $\beta = \Delta\lambda_z / \Delta\lambda_\theta$, and determined from experimental data. A Non-linear least squares method was used to determine the best-fit parameters of c, C_1, C_2, and C_3 for each specimen in both the circumferential and longitudinal directions.

RESULTS AND DISCUSSION: Stress-stretch relationships obtained from the theoretical model and experimental data were compared for both the circumferential and longitudinal directions (Figures 1&2). The modified Fung's model predicted the stress-stretch relationship in the physiological range for human renal arteries in different ages. The coefficients C_1, C_2 and C_3 were not significantly different up to 30 years old, and then increased remarkably with age ($p<0.05$) (Figure 3). The findings of mathematical simulation were in agreement with results of histology and mechanical testing [3]. C_1 (associated with the stiffness in the circumferential orientation) and C_2 (associated with the stiffness in the longitudinal orientation) have different values, reflecting that the arterial wall was anisotropic. Moreover, the increase of C_1 and C_2 implied the stiffening of the arterial wall due to degradation of elasin and deposition of collagen fibers, with increasing age. The increase C_3 with age demonstrated the strengthened interaction between the two orientations. A possible explanation is that newly deposited collagen fibers are randomly oriented, replacing the well aligned elastin fibers. Further study is necessary to examine the orientation of deposited collagen fibers with age.

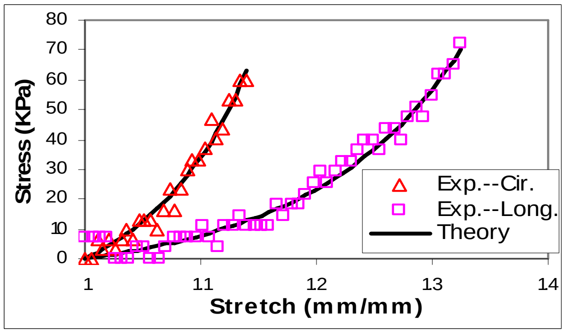

Figure 1: Mathematical simulation and experimental data (13 year old artery)

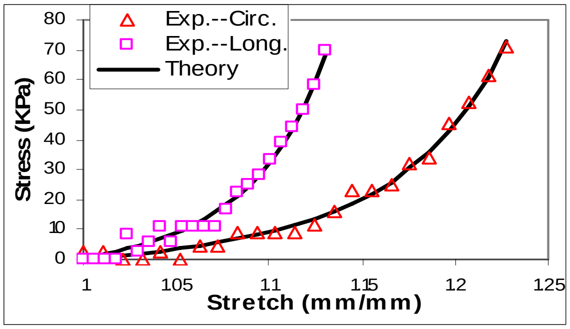

Figure 2: Mathematical simulation and experimental data (45 year old artery)

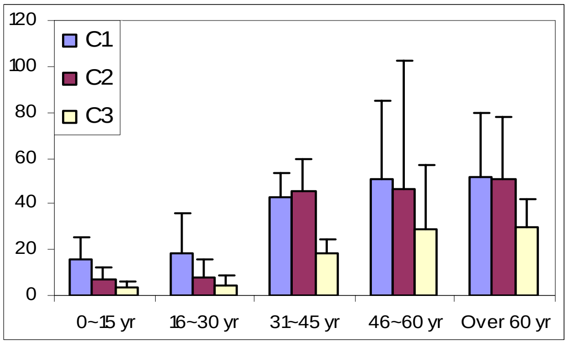

Figure 3: Average values for C_1, C_2 and C_3

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