Constitutive Model for Stress on Nickel – Titanium Shape Memory Alloy Considering an Austenite – Twinned Martensite – Detwinned Martensite Phase Transformation Approach

Manuel I Varela-Jiménez, Miguel Bueno, Jorge A Cortes.
Tecnológico de Monterrey, Campus Monterrey.

Statement of Purpose: Nickel – Titanium (NiTi) is a shape memory alloy also known as Nitinol. It has found application in medical stents for implantation in the human body due to its good corrosion resistance in physiological environments [1].The shape memory effect (SME) is a controlled transition between the austenite phase and the martensite structure [2]. Martensite has a different cubic lattice than austenite; thus the atomic repositioning causes the SME [3]. Several studies have been made about the NiTi but it exists the need of a numerical analysis of the phenomenology of the material since there are not mathematical models taking into account on a same equation its microstructure, stress, strain and temperature. This kind of model is required for Finite Element Analysis of NiTi devices leading to an optimal design. The objective of this work is to develop a constitutive model for three microstructural changes: 1) austenite to twinned martensite (temperature induced by cooling), 2) twinned martensite to detwinned martensite (strain induced) phase transformation, 3) detwinned martensite to austenite (temperature induced by heating); relating them with the stress behavior of NiTi.

Methods: The constitutive model is based on previous work on stainless steels with strain induced martensite phase transformation [4] which has been adapted for NiTi through of experiments of microstructural changes due to strain and heating of NiTi wires with a diameter of 0.1, 0.2 and 0.3 mm. Heating is done by connecting the wires to electric current; strain is applied by the use of a tensile test machine. First, the wires are characterized by determining the electric current – temperature relation and by metallographic analysis after the material has been strained or heated. Then, isothermal tensile tests are carried out in order to determine the stress of the austenite and martensite phases. Finally, the relation between austenite/martensite volume fraction and temperature is determined by measuring the length recovery of strained wires, assuming that at high temperature and maximal strain NiTi is fully austenitic and that at room temperature after strain recovery NiTi is fully martensitic.

Results: Wires require around 1 ampere to achieve 100°C. From metallographic analysis and tensile tests the transformation diagram at figure 1 is proposed; it shows that the material has a twinned martensitic structure at room temperature, it transforms into austenite when electric current is applied, however if NiTi is subjected to strain, twinned martensite transforms into detwinned martensite; these phase transformations follow a sigmoidal behavior and the stress depends of the volume fraction of the microstructures present under given under certain strain/temperature conditions. The overall thermo-mechanical behavior is described by the constitutive model at equation (1). It is based on an energy criterion which defines that the energy consumed to deform the austenite (*a*), twinned martensite (*tM*) or detwinned martensite (*dM*) phases in the system is equivalent to the energy consumed to deform the aggregate (*NiTi*).

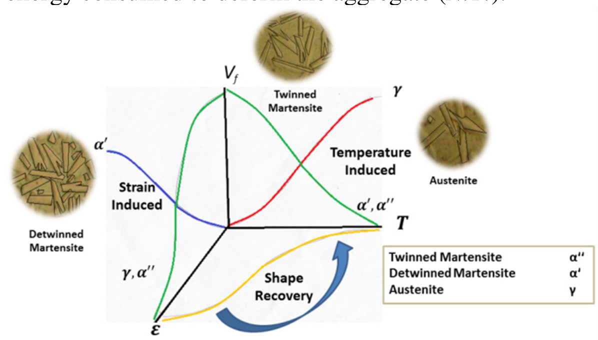

Figure 1. NiTi Phase Transformation Diagram

$$\sigma_{NiTi} = \sigma_A \cdot V_{fA} + \sigma_{mT} \cdot V_{fmT} + \sigma_{mD} \cdot V_{fmD} \quad (1)$$

The sum of the volume fraction V_f values is 1, these are described by equations (2) and (3)

$$V_{fA} = \left[1 + \left(\frac{T}{T_c}\right)^B\right]^{-1} \quad V_{fmD} = \left[1 + \left(\frac{\varepsilon}{\varepsilon_c}\right)^B\right]^{-1} \quad (2)$$

$$V_{fmD} = 1 - V_{fA} \qquad V_{fmD} = 1 - V_{fMD} \quad (3)$$

B is a material constant, T_c and ε_c are the characteristic temperature and characteristic strain on which 50% of the phase transformation is achieved; these are described by equations (4) and (5). According to equations (2) and (3), austenite is temperature induced while detwinned martensite is strain induced and the 3 phases does not coexist at any instant. From isothermal tensile tests, the stress for each one of the phases is determined through equations (6), (7) and (8).

$$\varepsilon_c = G \cdot e^{H \cdot T} \quad (4)$$

$$Ic = A \cdot e^{Qd} \quad (5)$$

$$\sigma_a = K_a \cdot \varepsilon^{n_a} \quad (6)$$

$$\sigma_{mT} = K_{mT} \cdot \varepsilon^{N_{mT}} \quad (7)$$

$$\sigma_{mD} = K_{mD} \cdot \varepsilon^{N_{mD}} \quad (8)$$

A, G, H, K, N and Q are material constants related to the wire diameter and the electric current that is applied.

Conclusions: Since the model shows good fitting between experimental and theoretical data, the assumptions and experimental work done result valid and the constitutive model based on an energy criterion including temperature and strain can successfully relate the microstructural characteristics of NiTi with its macroscopic behavior. It is an equation that requires easy inputs such as geometry and conditions of the material. As future work it could be applied for simulation of NiTi devices by Finite Element Analysis.

References: [1] Favier D. Mat Sci Eng A-Struct; 2006; 429: 130-136, [2] De Castro J A. Smart Mater Struc. 2007; 16: 2080-2090, [3] Nemat-Nasser S. Mech Mater. 2006; 38: 463-474, [4] Cortes J. *JSME Int J. 1992;* I 35 2: 201-209.